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Quantum Probabilities, Kolmogorov Probabilities, and Informational Probabilities

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The relations between quantum probabilities, Kolmogorov probabilities, and informational probabilities are studied against the background offered by the concept of a quantum mechanical probability tree built in previous work. It is shown that the quantum mechanical transformation theory goes beyond the Kolmogorov concept of probabilities. It is furthermore shown that the quantum mechanical concept of probability is of the same essence as the informational one. The analyses that produce these conclusions bring forth the first lines of a general mathematical representation of the emergence and circulation of patterns of any kind.

1. INTRODUCTION

Many authors, starting with Mackey (1963), have perceived that the quantum mechanical concept of probability has specifics that distinguish it from the Kolmogorovian one. In particular, detailed mathematical characterizations of such specifics have been given in recent work by Accardi (1983), Gudder and Zanghi (1984), Pitowski (1989), and Beltrametti and Maczynski (1991). In what follows I give another sort of characterization, basically *semantic*, where, starting from the quantum mechanical transformation theory, quantum probabilities are compared to both Kolmogorov probabilities and the theory of information.

In previous work (Mugur-Schächter, 1991, 1992*a*,*b*, 1993) I have studied the *space-time* organization of quantum probabilities. I have shown that this organization brings in centrally a certain type of treelike structure that I have called the "probability tree of a state preparation." Here, after a brief introduction to this new concept, I first show that it permits one to

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grasp intuitively the essential conceptual difference between Kolmogorov probabilities and quantum probabilities and to specify a formal signature of these. Then I show that the quantum mechanical probability trees can be imbedded in the theory of information, where one can construct an informational representation of the quantum mechanical transformation theory that renders this theory intelligible, both physically and conceptually. Thereby there will appear the first lines of a new formalism where the probabilistic, the quantum mechanical, and the informational representations combine.

In domains where formalisms have been strongly developed and play a crucial role it is not easy to draw attention to traits that go beyond the formalism, that have to be seized with its help, but traversing it to reach beneath. In order to free from any parasite the conceptual contours that we want to convey, we shall stick to most current mathematical expressions, such as can be found in textbooks.

The key concept in this work is that of a random phenomenon.

Before Kolmogorov's approach, the formalized features of the theory of probabilities concerned quasi-exclusively the probability measures. The involved events (elementary or not) were left in the substratum of the formalization and in general were not even symbolized. Kolmogorov has drawn them into the realm of the systematically symbolized and he has explicitly tied them to mathematical expression. In his approach any probability measure π is defined only inside a probability space $[U, \tau, \pi]$, where $U = \{e_i\}$ ($i \in I$, I an index set) is a universe of elementary events e_i, τ is an algebra of events defined on U and π is a probability measure posed on τ. Thereby it became obvious that a probability measure separated from a definite universe U of elementary events and a definite algebra τ of events on U is *not* a definite concept, that it is a fragment of a concept. This has been an enormous progress. However, the process of making explicit the whole concept of probability is not yet exhausted. In Kolmogorov's approach any universe of elementary events U is conceived to be produced by some "random phenomenon," but this notion is neither explicitly defined nor systematically symbolized. So the way in which the operations and processes from the random phenomenon produce the elementary events $e_i \in U$ remains vague. The structure of the channels through which semantic substance is drawn out from the pool of "physical reality" and is poured into the considered probability space is not yet specified, while on the other hand, though hidden, it plays a crucial role in the conceptualization. This entails, concerning the generated events and probability measures, very puzzling ambiguities. These motivated the *a posteriori* mathematical and logical characterizations of structures of probability measures and events quoted before (Accardi, 1983; Gudder and Zanghi, 1984; Pitowski, 1989; Beltrametti and Maczynski, 1991). Such ambiguities, however, can also be attacked from an *a priori* viewpoint, genetically, from beneath, by the physical study of the random phenomena. It can be hoped that the results obtained along these opposed directions will merge into a more complete and controlled view.

To begin with, in what follows the random phenomenon that produces the universe of elementary events from a considered probability space, quite systematically, will be explicitly symbolized. We shall indicate it by a pair (P, U), where P designates an "identically" reproducible procedure, each one realization of which brings forth one elementary event $e_i \in U$, in general variable from one realization of P to another one, notwithstanding the supposed identity of the reiterations, whereby, randomly, there emerges the whole universe U. Furthermore, in order to emphasize that each probability space is necessarily tied to some random phenomenon, we introduce the concept of a "probability *chain*" where the considered probability space is always preceded by the symbol of the corresponding random phenomenon:

$$(P, U) \rightsquigarrow [U, \tau, \pi] \tag{1}$$

One probability chain is the *minimal* autonomous and closed abstract probabilistic concept. The probability chains (1) are *indivisible* abstract molds. A probability space without a definite corresponding random phenomenon still is not a definite concept, it still is only a fragment of a concept, just like a probability measure without a definite universe of elementary events and a definite algebra of events on this.

But mere systematic symbolization of the random phenomenon is not sufficient, of course. The structure of what is indicated by the symbol (P, U) has to be specified also. In another work, within a "general syntax of relativized conceptualization," we have done this in quite general terms (Mugur-Schächter, 1991, pp. 277–286) and have drawn interesting consequences. Here, however, for simplicity, we adopt a less general approach, concerning specifically the quantum mechanical random phenomena. This will suffice for indicating the essence of the texture that relates Kolmogorov probabilities, quantum probabilities, and informational probabilities. Immersion into the general syntax of relativized conceptualization and a detailed logical and mathematical representation of the obtained structures will be attempted elsewhere.

2. THE QUANTUM MECHANICAL PROBABILITY TREES

2.1. The Essence of the Hilbert–Dirac Formulation of Quantum Mechanics

Quantum mechanics studies states of microsystems. These are represented by normalized kets $|\psi\rangle$ that are postulated to form a (Hilbert) vector space. From a physical point of view this formal postulate constitutes "the principle of superposition": If there "exist" two states with state vectors $|\psi_1\rangle$ and $|\psi_2\rangle$, then there also "exists" any state with state vector $|\psi_{12}\rangle = \lambda_1|\psi_1\rangle + \lambda_2|\psi_2\rangle$, where λ_1, λ_2 are arbitrary complex numbers.

The predictive qualifications of the states are probabilistic. Formally, they are obtained with the help of linear and in general noncommuting dynamical operators Ω (quantum mechanical observables). The language and the algorithms are as follows. It is asked, for instance, what (density of) probability $\pi(\psi, \omega_j)$ there is, if a measurement of the physical quantity represented mathematically by the observable Ω is performed on the state with state vector (ket) $|\psi\rangle$, to obtain a physical outcome V_j corresponding to the eigenvalue ω_j of Ω. The answer is founded on the principle of spectral decomposability (the expansion postulate): on the basis of this principle one performs the spectral decomposition of $|\psi\rangle$ with respect to Ω, $|\psi\rangle = \sum_j c(\psi, \omega_j)|u_j\rangle$, where $|u_j\rangle$ and ω_j are, respectively, the eigenvectors and the eigenvalues of Ω, determined by the equation $\Omega|u_j\rangle = \omega_j|u_j\rangle, j \in J$, J an index set, and $c(\psi, \omega_j) = \langle u_j|\psi\rangle$ are the expansion coefficients. The sought probability is postulated to be $\pi(\psi, \omega_j) = |\langle u_j|\psi\rangle|^2$ (contrary to certain beliefs, this cannot be entirely derived).

Two distinct predictive probability measures corresponding to two noncommuting observables Ω_1 and Ω_2 but to the same state vector $|\psi\rangle$ are related according to the quantum mechanical transformation theory, i.e., according to the equations $c(\psi, \omega_{2n}) = \sum_j \alpha_{nj} c(\psi, \omega_{1j})$, $j \in J$, $n \in N$ (J, N are index sets for the eigenvalues of, respectively, Ω_1 and Ω_2; $\alpha_{nj} = \langle v_n|u_j\rangle$ are the transformation coefficients from the eigenvectors $|u_j\rangle$ of Ω_1 to those $|v_n\rangle$ of Ω_2).

When the probability postulate $\pi(\psi, \omega_j) = |c(\psi, \omega_j)|^2$ is combined with the transformation formula $c(\psi, \omega_{2n}) = \sum_j \alpha_{nj} c(\psi, \omega_{1j})$ (as is done, in particular, within the representations of "successive measurements"), the transformed expression acquires the form

$$|c(\psi, \omega_{2n})|^2 = \left|\sum_j \alpha_{nj} c(\psi, \omega_{1j})\right|^2$$

$$= \sum_j |\alpha_{nj}|^2 |c(\psi, \omega_{1j})|^2 + [\text{"interference" terms}] \qquad (2)$$

where the "interference" terms contain products $c^*(\psi, \omega_{1j})c(\psi, \omega_{2k})$ or

$c(\psi, \omega_{1j})c^*(\psi, \omega_{2k})$: In the probability $\pi(\psi, \omega_{2n}) = |c(\psi, \omega_{2n})|^2$ of an eigenvalue of the observable Ω_2 *written as a function of the expansion coefficients corresponding to the observable* Ω_1, there appear terms involving the probabilities $\pi(\psi, \omega_{1j}) = |c(\psi, \omega_{1j})|^2$ of the eigenvalues of the observable Ω_1 *and* furthermore other terms. A probability $\pi(\psi, \omega_{2n})$ is not just a linear superposition of the probabilities $\pi(\psi, \omega_{1j})$. In this sense, there exists an "interference of probabilities" *tied to a change of representation* performed according to the transformation theory.

When the probability postulate $\pi(\psi, \omega_j) = |c(\psi, \omega_j)|^2$ is combined with expressions of the type $|\psi_{12}\rangle = \lambda_1|\psi_1\rangle + \lambda_2|\psi_2\rangle$ entailed by the principle of superposition, the probability $|c(\psi_{12}, \omega_j)|^2$ of an eigenvalue of the observable Ω acquires the expression

$$|c(\psi_{12}, \omega_j)|^2 = |\lambda_1 c(\psi_1, \omega_j) + \lambda_2 c(\psi_2, \omega_j)|^2$$
$$= |\lambda_1 c(\psi_1, \omega_j)|^2 + |\lambda_2 c(\psi_2, \omega_j)|^2 + [\text{"interference" terms}] \qquad (3)$$

So again the probability $\pi(\psi_{12}, \omega_j)$ is not just a linear superposition of the probabilities $\pi(\psi_1, \omega_j)$ and $\pi(\psi_2, \omega_j)$; there appear "interference" terms involving this time products $c(\psi_1, \omega_j)c^*(\psi_2, \omega_j)$ or $c^*(\psi_1, \omega_j)c(\psi_2, \omega_j)$. In this sense there exists also an "interference of probabilities" *tied to the principle of superposition.*

There are obvious formal similarities between these two sorts, (2) and (3), of "interference of probabilities." But there are also obvious *formal* differences (Mugur-Schächter, 1991, pp. 1414–1415). The existence of the latter is quite generally neglected. *A fortiori* no questions are raised concerning their semantic substratum. And, on the sole basis of the formal similarities, in both cases the situation is indicated in a flattening way by the same expression, "interference of probabilities." The interference of probabilities is the most striking specific of quantum theory as compared with the other probabilistic physical theories.

So vectors, operators, equations, and probability measures are manipulated accordingly to algorithms. Hidden beneath these algorithms, the probabilistic *organization* of the quantum theory remains obscure. What are the *correspondences* between, on the one hand, the basic quantum mechanical descriptors—state vectors $|\psi\rangle$, operators Ω, eigenfunctions and eigenvectors of these—and, on the other hand, the basic probabilistic descriptors, random phenomena and probability spaces? In the absence of clear answers the involved *significances* cannot be perceived. The physical meaning of the two principles that dominate the formalism—the principle of superposition and the principle of spectral decomposition—as well as the semantic content of the interferences of probabilities, remain vague.

We have shown (Mugur-Schächter, 1991, 1992*a*) that the correspondences between quantum mechanical and probabilistic descriptors can be established, and that indeed they enlighten the semantics encapsulated in the quantum mechanical formalism. The summary of these results is as follows.

2.2. Formal Quantum Mechanical Probability Chains

Consider a pair $(|\psi\rangle, \Omega)$, where $|\psi\rangle$ is the state vector assigned at the time t to the considered microsystem S and Ω is a Hermitian operator representing a quantum mechanical dynamical observable. For each such pair the quantum theory defines a family of elementary probability densities $\pi(\psi, \omega_j) = |\langle u_j|\psi\rangle|^2, j \in J$ (J an index set) for the emergence of an eigenvalue ω_j of the observable Ω when a measurement of Ω is performed on S in the state $|\psi\rangle$. These generate a definite probability for any event constructed from elementary events ω_j. So one can define for any pair $(|\psi\rangle, \Omega)$ a formal "probability chain," i.e., a sequence (random phenomenon) $\rightsquigarrow$ [a probability space] that can be symbolized by

$$[(\psi, \Omega), \{\omega_j\}] \rightsquigarrow [\{\omega_j\}, \tau, \pi(\psi, \Omega)] \tag{1'}$$

where $[(\psi, \Omega), \{\omega_j\}]$ is the symbol for the random phenomenon that involves the state vector $|\psi\rangle$ and the dynamical observable Ω and produces, by reiteration, the universe $\{\omega_j\}$ of formal elementary events: τ is the *total* algebra on $\{\omega_j\}$; and $\pi(\psi, \Omega)$ is the probability density law on τ determined, via the law of total probabilities, by the elementary probability density law $\pi(\psi, \omega_j) = |\langle u_j|\psi\rangle|^2$. The set of all the formal chains (1′) with total algebras inside them includes the expressions of all the conceivable quantum mechanical predictions.

2.3. Factual Quantum Mechanical Probability Chains

Each formal probability chain (1′) points toward a corresponding factual probability chain

$$[(P_\psi, M_\Omega), \{V_{\Omega j}\}] \rightsquigarrow [\{V_{\Omega j}\}, \tau_F, p(P_\psi, M_\Omega)] \tag{1''}$$

where P_ψ is the operation of state preparation that produces the state with state vector $|\psi\rangle$; M_Ω is an *individual* measurement evolution corresponding to the observable Ω; $V_{\Omega j}$ is a "needle position" of a macroscopic device D_Ω for measurements of the observable Ω; $[(P_\psi, M_\Omega), \{V_{\Omega j}\}]$ is the random phenomenon that involves the operation P_ψ of state preparation and the individual measurement evolutions M_Ω and which, by reiteration, produces the universe of elementary events $\{V_{\Omega j}\}$; τ_F is the total algebra on $\{V_{\Omega j}\}$ (F: factual); and $p(P_\psi, M_\Omega)$ is the probability measure on τ_F.

2.4. The Factual Quantum Mechanical Random Phenomena

So:

- *A quantum mechanical random phenomenon consists of a sequence of two distinct operations, an operation P_ψ of state preparation and a measurement operation M_Ω that ends with the registration of a "needle position" $V_{\Omega j}$.*

This is a complex structure of which the characteristics and their consequences will progressively appear in what follows.

2.5. Connection between Formal and Factual Chains

The probability chains (1′) and (1″) are connected as follows. Each eigenvalue $\omega_j \in \{\omega_j\}$ from a formal chain is posited to be calculable as a function $\omega_j = f_\Omega(V_{\Omega j})$ of that observed "needle position" $V_{\Omega j}$ from the factual chain that is labeled by the same index $j \in J$. Furthermore, each factual elementary probability density $p(P_\psi, M_\Omega, V_{\Omega j})$ from $p(P_\psi, M_\Omega)$ is posited to generate—via $\omega_j = f_\Omega(V_{\Omega j})$—the corresponding formal elementary probability density $\pi(\psi, \Omega, \omega_j)$ contained in $\pi(\psi, \Omega)$:

$$p(P_\psi, M_\Omega, V_{\Omega j}) \Rightarrow \pi(\psi, \Omega, \omega_j) = |\langle u_j|\psi\rangle|^2$$

This—and only this—designates the family of assertions that quantum mechanics offers for experimental testing (Mugur-Schächter, 1992*b*).

2.6. Elementary Quantum Mechanical Chain Experiments

A sequence P_ψ-M_Ω-$V_{\Omega j}$ from a quantum mechanical random phenomenon that generates a factual chain (1″) will be called an "elementary quantum mechanical chain experiment" (eqmce). It possesses a remarkable *unobservable* operational-processual *depth* wherefrom there emerges into the observable *only* the extremity $V_{\Omega j}$ that contributes to the construction of the factual observable universe of elementary events $\{V_{\Omega j}\}$ from a chain (1″). Each observable quantum mechanical "event" (*non*elementary) from an algebra τ_F from a factual quantum mechanical probability space contains inside its semantic substratum all the unobservable sequences of operations and processes forming the corresponding elementary quantum mechanical chain experiments that end up with the registrations $V_{\Omega j}$ of a needle position contained in that event. So *any* quantum mechanical prediction concerns either only one elementary quantum mechanical chain experiment or a whole union of such experiments.

- *The elementary quantum mechanical chain experiments from a quantum mechanical random phenomenon are the "fibers" out of which is* *made the factual substance of the quantum theory. They generate directly the basic, the* individual *level of the quantum mechanical probabilistic conceptualization, reflected in the universe* $\{V_{\Omega j}\}$ *of elementary events.*

Therefrom are then successively built the other two—abstract—metalevels of conceptualization, the statistical metalevel, reflected in the algebra τ_F of events, and the probabilistic meta-metalevel, reflected in the probability measure $p(P_\psi, M_\Omega)$.

The individual fibers P_ψ-M_Ω-$V_{\Omega j}$ specify completely the structure of the physical-operational channels by which semantic substance, drawn from the pool of what is called physical reality, is transformed and poured into the factual quantum mechanical probability spaces. So also into the formal ones, via the two connective relations

$$\omega_j = f_\Omega(V_{\Omega j}), \qquad \pi(\psi, \omega_j) = |\langle u_j | \psi \rangle|^2 = \pi(P_\psi, M_\Omega, V_{\Omega j})$$

Nonetheless, and this is a striking fact:

- *Within the formalism of quantum mechanics the individual, elementary quantum mechanical chain experiments are* devoid of any representation.

Correlatively, the formalism does not distinguish clearly between the three different levels of conceptualization that are involved, the individual level, the statistical level, and the probabilistic level. This introduces much confusion, especially in the logical approaches (Mugur-Schächter, 1992*b*).

2.7. The Quantum Mechanical Probability Trees

We *fix* now an operation of state preparation P_ψ. Consider the set of all the *factual* probability chains (1″) determined by P_ψ *and* the set of *all* the physical processes M_Ω of measurement evolution corresponding to all the dynamical observables $\Omega_1, \Omega_2, \Omega_3, \ldots$ defined in quantum mechanics. The probability chains from this set constitute together a certain *whole*, a certain unity, in consequence of their common provenance P_ψ.

What is the *space-time* structure of this unity?

This can be regarded as the central question of the present inquiry into quantum probabilities.

For all the chains from the considered unity, the space-time support of the operation of state preparation P_ψ is by definition the *same*. But not also for all the individual measurement evolutions M_Ω involved in this unity. The set of the individual measurement evolutions *splits* into classes $M_X, M_Y, \ldots$, each class of measurement evolutions corresponding to a set $\{\Omega_X^h, h = 1, 2, \ldots, m\}$, $\{\Omega_Y^h, h = 1, 2, \ldots, k\}, \ldots,$ of $m, k, \ldots,$ mutually "compatible" observables, whereas two observables from sets of observables tied to two different classes of measurement evolutions are mutually incompatible.

Many textbooks as well as some research papers contain very confusing considerations and mathematical nuances concerning "successive or simultaneous measurements of compatible observables" versus the projection postulate, formal commutation, etc. But in fact the qualifications "simultaneous" or "successive" simply are *devoid of relevance* with respect to the physical features involved in the concept of compatible observables. Indeed consider first only one class of individual measurement evolutions, say M_X. Each *one* measurement evolution from M_X can be constructed—physically—such that the *one* registration of *one* value V_{Xj} of the "needle position" of the corresponding macroscopic device D_X produced by it permits one to *calculate*—from this unique physical datum, but with the help of *various* theoretical connecting definitions $\omega^h_{Xj}=f^h_{Xj}(V_{Xj})$, $h=1,2,\ldots,m$—*all* the m *different* eigenvalues ω^h_{Xj} labeled by the *same* index j, for all the observables Ω^h_X from the set of mutually compatible observables corresponding to the class M_X of measurement evolutions. (Formally, this amounts to mathematical constructibility, out of Ω^h_X, of any other observable $\Omega^{h'}_X$ from this same class M_X, and to commutativity of Ω^h_X and $\Omega^{h'}_X$). So each set of m eigenvalues ω^h_{Xj}, $h=1,2,\ldots,m$, produced by one registration V_{Xj}, involves just *one* physical process of individual measurement evolution M_X, covering just one given space-time support. No condition of (physical) simultaneity or successivity is involved.

Consider now two distinct classes M_X and M_Y. These, by definition, are "mutually incompatible" in the sense that it is *not* possible to construct physically an individual measurement evolution such that the unique outcome V_{Xj} produced by it permits calculation of both a corresponding eigenvalue tied with M_X and a corresponding eigenvalue tied with M_Y. *This is what is commonly called "Bohr complementarity."* Formally, this impossibility is expressed by mathematical nonconstructibility of an observable Ω_X corresponding to the class M_X from an observable Ω_Y corresponding to another class M_Y, and by noncommutativity between Ω_X and Ω_Y.

The situation entails that, globally, the whole, the unity constituted by the ensemble of all the factual probability chains corresponding to a fixed operation of state preparation P_ψ possesses *a branching, a treelike space-time structure.* Let us symbolize this treelike structure by $\mathbf{T}(P_\psi)$ and let us call it "the quantum mechanical probability tree of the operation of state preparation P_ψ."

Figure 1 provides a simplified example of a probability tree with only four observables: two compatible observables Ω^a_{12} and Ω^b_{12} corresponding to the same class M_{12} of individual measurement evolutions, and two

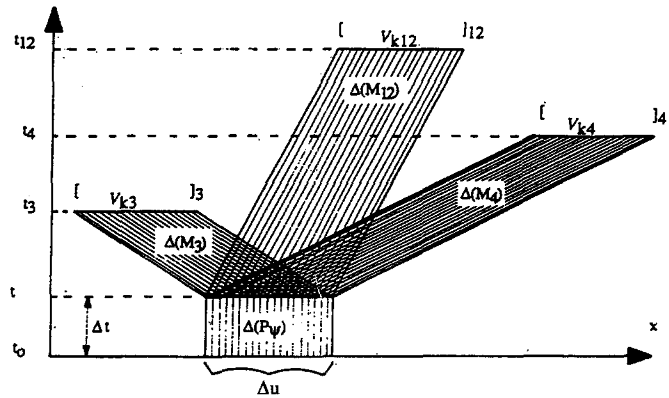

Fig. 1. The quantum mechanical probability tree $\mathsf{T}(P_\psi)$ of the operation of state preparation P_ψ.

incompatible observables Ω_3 and Ω_4 tied, respectively, to individual measurement evolutions M_3 and M_4. The factual, observational probability spaces corresponding to the measurement evolutions M_{12}, M_3, and M_4 realized on the state represented by $|\psi\rangle$ are indicated respectively by the notations $[V_{k12}]$, $[V_{k3}]$, and $[V_{k4}]$. Each one of the probability spaces $[V_{kn}]$, $n = 1, 2, 3, 4$, emerges—with respect to an origin of times *reset to zero after each elementary quantum mechanical chain experiment*—at some corresponding specific time t_{12} (i.e., $t_{12} - t$), t_3 (i.e., $t_3 - t$), and t_4 (i.e., $t_4 - t$). The branch corresponding to Ω^a_{12} and Ω^b_{12}, so to M_{12}, contains a very big number of fibers P_ψ-M_{12}-V_{12j} each one of which ends up with *one* needle position, say $V_{12j'} \in \{V_{12j}\}$, that permits us to calculate *two* distinct corresponding eigenvalues, $\omega^a_{12j'} \in \{\omega^a_{12j}\}$ and $\omega^b_{12j'} \in \{\omega^b_{12j}\}$, via two different theoretical definitions, $\omega^a_{12j'} = f^a_{12}(V_{12j'})$, $\omega^b_{12j'} = f^b_{12}(V_{12j'})$. The branch corresponding to Ω_3, so to M_3, contains a big number of fibers P_ψ-M_Ω-V_{3j} each one of which ends up with a needle position $V_{3j'} \in \{V_{3j}\}$ that permits one to calculate a *unique* corresponding eigenvalue $\omega_{3j'} \in \{\omega_{3j}\}$ via a theoretical connecting function $\omega_{3j'} = f_3(V_{3j'})$. Similarly the branch corresponding to Ω_4, so to M_4, contains a big number of fibers P_ψ-M_Ω-V_{4j} each one of which ends up with a needle position $V_{4j'} \in \{V_{4j}\}$ that permits one to calculate a *unique* corresponding eigenvalue $\omega_{4j'} \in \{\omega_{4j}\}$ via a theoretical definition $\omega_{4j'} = f_4(V_{4j})$. So the space $[V_{k12}]$ is endowed with more specifications than the spaces $[V_{k3}]$ and $[V_{k4}]$.

In all the fibers of the tree the initial phase, of state preparation, covers the same space-time domain $\Delta(P_\psi) = \Delta x \Delta t$ (the common space-time trunk of the tree), with $\Delta t = t - t_0$. In the case of an evolution M_{12} corresponding to the two commuting observables Ω^a_{12} and Ω^b_{12}, the subsequent phase of measurement evolution covers, for each fiber corresponding to the process M_{12} of measurement evolution, a unique space-time domain $\Delta(M_{12})$. In the case of a measurement evolution M_3 corresponding to the observable Ω_3 or a measurement evolution M_4 corresponding to the observable Ω_4, the involved space-time domains are distinct, namely $\Delta(M_3)$ and $\Delta(M_4)$.

So in the concept of probability tree of a state preparation P_ψ, the individual, the elementary quantum mechanical chain experiments are explicitly represented and they *determine* the corresponding branching space-time structure of the tree. The other two involved levels of conceptualization—the statistical and the probabilistic—are generated by two distinct and hierarchically connected sorts of reiterations (Mugur-Schächter, 1991, 1992*a*, 1992*b*) and they are implied only in the probability spaces from the tops of the branches, namely, respectively, in the algebras and in the probability measures from these spaces.

A quantum mechanical probability tree is a remarkably comprehensive construct. Most of the fundamental algorithms of the quantum mechanical calculus which combine *one* normed state vector with the dynamical operators representing the quantum mechanical observables can be defined *inside* any *one* tree $\mathbf{T}(\psi)$:

1. The mean value of an observable Ω in a state with vector $|\psi\rangle$: $\langle\psi|\Omega|\psi\rangle$, $\forall|\psi\rangle$, $\forall\Omega$.

2. The uncertainty theorem, for any pair of observables:

$$\langle\psi|(\Delta\Omega_1)^2|\psi\rangle\langle\psi|(\Delta\Omega_2)^2|\psi\rangle \geq |\langle\psi|(i/2)(\Omega_1\Omega_2-\Omega_2\Omega_1)|\psi\rangle| = (1/2)(h/2\pi), \qquad \forall\Omega_1, \Omega_2$$

3. The principle of spectral decomposability (expansion postulate)

$$|\psi\rangle = \sum_j c(\psi, \omega_j)|u_j\rangle, \qquad \forall|\psi\rangle, \quad \forall A\colon \quad A|u_j\rangle = \omega_j|u_j\rangle$$

[$c(\psi, \omega_j)$ are the expansion coefficients] which permits us to calculate the probability density $\pi(\psi, \omega_j)$ via the probability postulate

$$\pi(\psi, \omega_j) = |\langle u_j|\psi\rangle|^2 = |c(\psi, \omega_j)|^2$$

4. Finally, the whole quantum mechanical "transformation theory" from the basis $\{|u_j\rangle\}$ of an observable Ω_1 with eigenvalues ω_{1j} to the basis $\{|v_k\rangle\}$ of an observable Ω_2 that does not commute with Ω_1 and has eigenvalues ω_{2j} (so from a branch with measurement evolutions M_1 to that with measurement evolutions M_2):

$$c(\psi, \omega_{2k}) = \sum_j \alpha_{kj} c(\psi, \omega_{1j})$$

$$\forall\Omega_1, \Omega_2\colon \quad \Omega_1|u_j\rangle = \omega_{1j}|u_j\rangle \quad \text{and} \quad \Omega_2|v_k\rangle = \omega_{2k}|v_k\rangle, \quad \forall j \in J, \quad \forall k \in K$$

(J, K are index sets for the eigenvalues of, respectively, Ω_1, Ω_2; $\alpha_{kj} = \langle v_k | u_j \rangle$ are the transformation coefficients).

2.8. The Principle of Superposition: A Calculus with Whole Trees

But as soon as the principle of superposition comes into play the embeddability into one tree hits a limit. The corresponding algorithms cease to be embeddable into one single probability tree: *Several trees have to be combined.* The quantum mechanical formalism contains implicit calculi with *whole probability trees.*

The principle of superposition is connected with expressions of the type $|\psi_{12}\rangle = \lambda_1|\psi_1\rangle + \lambda_2|\psi_2\rangle$ that combine (at least) three trees, namely those introduced by the three operations of state preparation $P_{\psi 12}$, $P_{\psi 1}$, and $P_{\psi 2}$ corresponding to the three involved state vectors $|\psi_{12}\rangle$, $|\psi_1\rangle$, and $|\psi_2\rangle$. The acceptance of such *linear* composition expressions for any pair of functions ψ_1, ψ_2 is a condition *sine qua non* for the formal representability of the set of such functions by "kets" $|\psi_1\rangle$ and $|\psi_2\rangle$ that are abstract objects forming a *vector space.* However, and this is of basic importance, the state vectors $|\psi_{12}\rangle$, $|\psi_1\rangle$, and $|\psi_2\rangle$ themselves are only *indirectly* concerned in the principle of superposition:

- Regarded as a *physical* assertion, the principle of superposition concerns *directly* only the *operations of state preparation* $P_{\psi 1}$, $P_{\psi 2}$, and $P_{\psi 12}$ which produce the states with state vectors $|\psi_1\rangle$, $|\psi_2\rangle$, and $|\psi_{12}\rangle$ (Mugur-Schächter, 1991, pp. 1405–1424). Namely it amounts to the following assertion: *If the two operations* $P_{\psi 1}$, $P_{\psi 2}$ *are realizable separately, then also realizable is any operation* $P_{\psi 12}$ *that is some functional of these operations,* $P_{\psi 12} = G(\lambda_1, \lambda_2, P_{\psi 1}, P_{\psi 2})$, *such that it produces the state with state vector* $|\psi_{12}\rangle = \lambda_1|\psi_1\rangle + \lambda_2|\psi_2\rangle$.

On the other hand, the probability law for the state $|\psi_{12}\rangle$, for any observable Ω,

$$\begin{aligned}\pi(\psi_{12}, \omega_j) &= |\langle u_j|\psi_{12}\rangle|^2\\ &= |\langle u_j|\lambda_1\psi_1 + \lambda_2\psi_2\rangle|^2\\ &= \pi(\psi_1, \omega_j) + \pi(\psi_2, \omega_j) + [\text{"interference" terms}]\end{aligned}$$

"compares" the three probabilities $\pi(\psi_1, \omega_j)$, $\pi(\psi_2, \omega_j)$, and $\pi(\psi_{12}, \omega_j)$. Namely it *refers* the various probability measures $\{\pi(\psi_{12}, \omega_j), j \in J\}$, $\forall\Omega$, from the probability spaces of the *unique* tree obtained when the operation of state preparation $P_{\psi 12} = G(\lambda_1, \lambda_2, P_{\psi 1}, P_{\psi 2})$ is realized, to the corresponding probability measures $\{\pi(\psi_1, \omega_j), j \in J\}$, $\forall\Omega$, and $\{\pi(\psi_2, \omega_j), j \in J\}$, $\forall\Omega$, from the trees that *would* be obtained *if* the operations of state preparation $P_{\psi 1}$ and $P_{\psi 2}$ *were* realized *separately*. In short, the well-known and so puzzling algorithmic injunction "the amplitudes of probability have to be added, only the probabilities interfere" corresponds to the following distribution of the descriptional roles:

1. For the composition of the—nonobservable—state descriptors $|\psi\rangle$, a linear representation is chosen. *This permits a vector space formalism for the state descriptors*, which is highly convenient in calculations, but offers no possibility to express interaction between the physical effects of two separately realizable operations of state preparation $P_{\psi 1}$ and $P_{\psi 2}$ when these are involved in a more complex operation of state preparation $P_{\psi 12} = G(\lambda_1, \lambda_2, P_{\psi 1}, P_{\psi 2})$.

2. For the composition of the—observable—probability distributions corresponding to a state described by a linear combination of state vectors, a nonlinear representation is chosen. This permits one to express the observable, *factually existing* mutual "influences" between the physical effects of separately realizable operations of state preparation when these are involved together nonsequentially, "in parallel," in a more complex operation of state preparation that produces the state represented by the considered linear combination of state vectors.

This distribution of the descriptional roles is just a pragmatically convenient system of *choices of representation*. And it is most important to *distinguish* clearly between the formal features chosen for the representation of the studied facts and the physical characteristics of the facts themselves: *notwithstanding the use that it makes of vector spaces, quantum mechanics, by its quadratic definition of probabilities, is a nonlinear theory that describes in general nonlinear processes*. This is a noteworthy example of descriptional strategy.

2.9. Spectral Decomposition of One State Vector Versus Superposition of Several State Vectors

The concept of a quantum mechanical probability tree brings strikingly into evidence that the expressions of linear composition—in the mathematical sense—involved in the principle of spectral decomposition refer to facts that are *fundamentally different* from the expressions of linear composition involved in the principle of superposition:

1. The principle of spectral decomposition concerns future results of the *measurement* on an already prepared state with state descriptor $|\psi\rangle$, of the *one* dynamical observable tied to the considered decomposition: The root P_ψ and the trunk $|\psi\rangle$ of a single tree are given, and each one of the applications of the principle of spectral decomposition involves (is contained in) exclusively this *one* tree founded on this root-and-trunk

$(P_\psi, |\psi\rangle)$, the object of study being what is denoted by "$|\psi\rangle$." The principle of spectral decomposition involves *no reference to any other trees* (Mugur-Schächter, 1991, pp. 1412–1416).

2. Whereas, as already emphasized, the quantum mechanical principle of *superposition* concerns basically *operations of state preparation* $P_{\psi i}$, $i = 1, 2, \ldots, n$, so *roots* of *several* trees. Each one of these roots $P_{\psi i}$, $i = 1, 2, \ldots, n$, could found its own probability tree, but in fact this possibility is not realized, all the considered roots $P_{\psi i}$ being combined in only one effectively realized operation of state preparation $P_{\psi 12\ldots n} = G(\lambda_1, \lambda_2, P_{\psi 1}, P_{\psi 2}, \ldots, P_{\psi n})$ that founds only one tree; which, as mentioned, entails *reference* relations between the one effectively realized tree and the several other possible trees (Mugur-Schächter, 1991, pp. 1421–1424), in particular reference relations involving *all* the probability measures from these other trees, for *any* dynamical observable.

Besides these semantic mutual specificities, there are also purely mathematical specifics of the linear composition expressions involved in the principle of spectral decomposition, with respect to those involved in the principle of superposition (Mugur-Schächter, 1991, pp. 1412–1416).

2.10. Two Sorts of Interference of Probabilities

The fundamental difference between the physical significance of a spectral decomposition of one state vector and the physical significance of a superpsition of several state vectors splits the fundamental quantum mechanical notion of "interference of probabilities," into two essentially different sorts of interferences of probabilities (Mugur-Schächter, 1991, pp. 1412–1416).

1. The interferences of probabilities entailed by the transformation theory [equation (2)] involve the principle of spectral decomposition applied, for one state vector, to—necessarily—*two* bases introduced by two distinct and noncommuting observables Ω_1 and Ω_2; this sort of interference of probabilities is found to describe an *abstract*, only conceived "interaction" between the two—factually incompatible—predictional points of view (or grids) for future qualification of the studied state, corresponding to Ω_1 and to Ω_2. This first sort of interference of probabilities will be shown in the following sections to define the essential semantic difference between Kolmogorov probabilities and quantum probabilities, *and*, furthermore, to be a basic character of the informational approach.

2. Whereas the interferences of probabilities tied to superposition state vectors $|\psi_{12}\rangle = \lambda_1|\psi_1\rangle + \lambda_2|\psi_2\rangle$, can emerge for a *single* observable Ω (any one), and they are found, as mentioned, to describe *physical interactions*. If Ω is the position observable, these interactions, in certain conditions, are

even directly observable. Furthermore, the interference of probabilities tied to the superposition state vectors are found to involve nonremovably a very peculiar *model* for what is called a microsystem, notwithstanding the orthodox claim that any model is banished from the quantum theory (Mugur-Schächter, 1991, pp. 1429–1430). This second sort of interference of probabilities can be related to the basic specificities of nonsequential, "network," parallel computation as compared with the sequential Turing machine computations.

2.11. Confusing Mathematical "Unifications"

The Hilbert–Dirac formalism and language tend to *identify* the spectral decompositions of one state vector on a basis of eigenkets, with the superpositions of several normalized state vectors. *Ipso facto* they tend to identify also the interferences of probabilities entailed (within the transformation theory) by the principle of spectral decomposability, with the interferences of probabilities entailed by the principle of superposition. Such identifications cannot be regarded as a conceptual unification. They are just semantic *confusion*, within a flattening concept of linear composition—in a purely mathematical sense—of "generalized kets" from a Hilbert space of kets. This confusion has secreted an opaque stratum of conceptual mud where the "interpretation problems" have floundered for more than 60 years (Mugur-Schächter, 1993, pp. 98–121). *Furthermore*, it conceals crucial significances involved in the quantum mechanical formalism, the very germ of a unified mathematical representation of the emergence and evolution of patterns inside networks of "information processing entities" of any kind.

2.12. Global View

So any observable quantum mechanical elementary event $V_{\Omega j}$ is brought forth by some elementary quantum mechanical chain experiment, some fiber P_ψ-M_Ω-$V_{\Omega j}$. These fibers are the semantic matter described by the quantum theory. Any fiber P_ψ-M_Ω-$V_{\Omega j}$ belongs to a probability chain $(P_\psi, M_\Omega, \{V_{\Omega j}\}) \rightsquigarrow [\{V_j\}, \tau_F, p(P_\psi, M_\Omega, V_{\Omega j})]$. In its turn any probability chain belongs to a branch of a probability tree $\mathsf{T}(P_\psi)$, the tree tied to the operation P_ψ of state preparation which starts that chain. So the probability trees define a *partition* on the set of all the chains (hence on the set of all the fibers, hence on the set of all the observable quantum mechanical elementary events $V_{\Omega j}$).

When one contemplates the landscape determined by this partition each tree appears endowed with its own "internal" calculus (mean value of any dynamical observable Ω with respect to the state vector $|\psi\rangle$ tied to the considered tree, the uncertainty theorem for this state, the principle of spectral decomposition, the predictional probability laws for this state, and the whole quantum mechanical "transformation theory" that relates the probability measures from the different branches of the tree), while the different trees are related by a calculus with whole trees determined by the principle of superposition and the probability law for superposition states. This is a *hierarchical* view (fibers, chains, trees, connections between trees). It draws attention to the role played by the *space-time characteristics* of the operations by which the observer produces the objects to be studied (state preparations) and the processes of qualification of these (measurement operations).

How did the concept of a quantum mechanical probability tree emerge? We have performed just an attentive analysis of the connections between, on the one hand, *Kolmogorov's standard fundamental probabilistic concepts* (identically reproducible procedure, universe of elementary events, an algebra of events on this universe, a probability measure on this algebra) and on the other hand the main descriptors of the quantum mechanical formalism (state vectors, operators, eigenfunctions) and the factual counterparts of these. The *unique* novelty has been an explicit representation of the physical processes involved in the quantum mechanical random phenomena, with their space-time characteristics. And *this* novelty brought forth, with a sort of inner necessity, the probabilistic metaconstruct with treelike space-time support described above. But this metaconstruct, notwithstanding the fact that it has been produced by only a systematic confrontation with current standard probabilistic concepts, obviously *transcends* the abstract theory of probabilities as it now stands. Indeed, in Kolmogorov's approach the most complex basic probabilistic structure explicitly defined is *one* probability *space*. Not even the notion of a probability *chain* is explicitly defined as a basic monolithic construct. *A fortiori*, the concept of a probability tree, consisting of a whole family of irreducibly distinct but interconnected probability chains, a family of incompatible random phenomena rooted into the same operation of state preparation, is devoid of an abstract equivalent defined within the current theory of probabilities.

- *In Kolmogorov's approach, features induced by the space-time characteristics of the random phenomena have somehow been abstracted away.*

This first conclusion already suffices for proving the fundamental importance of an explicit use of the abstract concept of a probability chain, and of an explicit specification, in any application, of the physical structure and content of the involved random phenomenon, with its space-time characteristics. The structure and content of the random phenomenon that generates a probability space are the very *roots* of the probabilistic conceptualization. So far these roots have remained hidden. We shall now show that, uncovered, they bring into evidence the basic difference between Kolmogorov probabilities and quantum probabilities, and furthermore they reveal a basic unity between the quantum probabilities and the informational approach.

3. QUANTUM PROBABILITIES VERSUS KOLMOGOROV PROBABILITIES

3.1. "Deterministic Probabilistic Metadependence" between Branches of One Tree

It has been often emphasized that the quantum mechanical formalism does not define conditional probabilities relating, for a given state vector, the probabilities of eigenvalues of noncommuting observables. *Nevertheless*, and this is a striking fact, the quantum mechanical transformation theory [$c(\psi, \omega_{2k}) = \sum_j \alpha_{kj} c(\psi, \omega_{1j})$, $\forall \Omega_1, \Omega_2$: $\Omega_1 |u_j\rangle = \omega_{1j} |u_j\rangle$, $\Omega_2 |v_k\rangle = \omega_{2k} |v_k\rangle$, $\forall j \in J$, $\forall k \in K$, J, K index sets, $\alpha_{kj} = \langle v_k | u_j \rangle$ the transformation coefficients] permits us to determine *entirely* from the knowledge of the *whole* probability measure $\pi(\psi, \Omega_1)$ from one branch of a probability tree, any probability density $|c(\psi, \omega_{2k})|^2$ of any elementary event (so also of any event) involved in another branch of that tree tied to an observable Ω_2 which does not commute with Ω_1. So the quantum mechanical transformation theory is equivalent to the specification of a functional relation

$$\pi(\psi, \Omega_2) = F_{QM}[\pi(\psi, \Omega_1)] \tag{4}$$

between the two measures $\pi(\psi, \Omega_2)$ and $\pi(\psi, \Omega_1)$, which amounts to *coding* the measure $\pi(\psi, \Omega_2)$, initially expressed in its "native" language $\{|v_k\rangle, \omega_{2k}|\}$ introduced by the observable Ω_2 itself, into the "foreign" language $\{|u_j\rangle, \omega_{1j}|\}$ that is the "native" language of the measure $\pi(\psi, \Omega_1)$ concerning another observable Ω_1, incompatible with Ω_2.

The relation (4) can also be regarded as a "*deterministic probabilistic metadependence*" in the following sense: According to the current theory of probabilities the concept of "probabilistic dependence" is by definition confined inside *one* probability space where it concerns *isolated* pairs of *events*. Two events are tied by a "probabilistic dependence" if knowledge of one of these events "influences" the expectations concerning the other one. So the relation $\pi(\psi, \Omega_2) = F_{QM}[\pi(\psi, \Omega_1)]$ of mutual determination of the probability measures from a quantum mechanical probability tree can naturally be regarded as a "deterministic probabilistic metadependence":

"deterministic" because it consists in mutual *determination*; "probabilistic" because, though this determination is a certainty about "influence," nevertheless it concerns *probabilistic* constructs; "metadependence" because it concerns, not pairs of events from one space, but *globally* pairs of probability *measures* on entire algebras of events from incompatible probability spaces, which, with respect to events, are *meta* entities.

The notion of a probabilistic metadependence can also be upheld otherwise (Mugur-Schächter, 1992*b*, pp. 990–991). Imagine a physicist who does not yet know which state vector $|\psi\rangle$ "describes" the state produced by the operation of state preparation P_ψ. So he makes various measurements on this state in order to establish experimentally relative frequencies permitting one to induce the postulation of probability densities that shall determine an adequate mathematical descriptor $|\psi\rangle$ (Mugur-Schächter, 1991, pp. 1408–1412). Suppose that he decides to work with two noncommuting observables Ω_1 and Ω_2, and, on the basis of some reasons, envisages two *sets* of possible probability measures on the corresponding spectra, namely $\Sigma_1 = \{\pi(\psi, \Omega_1)\}$ and $\Sigma_2 = \{\pi(\psi, \Omega_2)\}$, respectively (for simplicity suppose they are discrete). The physicist now asks: What is the (meta)probability for finding, by measurements, this or that probability measure from Σ_1 or this or that probability measure from Σ_2?" In the absence of any criteria for answering otherwise, he will have to presuppose equipartition on both Σ_1 and Σ_2.

Suppose that, furthermore, the physicist has to answer the problem: "If for the spectrum $\{\omega_{m1}\}$ of Ω_1 the probability measure were $\pi_k(\psi, \Omega_1) \in \Sigma_1$ (k: known), what would be the corresponding *conditional* probability to find this or that measure $\pi(\psi, \Omega_2)$ from Σ_2 on the spectrum $\{\omega_{j2}\}$ of eigenvalues of Ω_2?" This new problem concerns now the product-probability space where the elementary events are all the possible associations $(\pi_k(\psi, \Omega_1), \pi(\psi, \Omega_2))$ between the one measure $\pi_k(\psi, \Omega_1) \in \Sigma_1$ (supposed known) and the various unknown probability measures envisaged in the set $\Sigma_2 = \{\pi(\psi, \Omega_2)\}$. Now, in the absence of any theory or data, the physicist, again, *must presuppose equipartition*—which amounts to presupposing *independence* between $\pi_k(\psi, \Omega_1)$ and the measures $\pi(\psi, \Omega_2) \in \Sigma_2$; that is, that the probability of a joint event $(\pi_k(\psi, \Omega_1), \pi(\psi, \Omega_2))$ is the *product* of the probability of the known measure $\pi_k(\psi, \Omega_1)$ (fixed) and of the probability of the unknown measure $\pi(\psi, \Omega_2)$ (variable inside Σ_2 and there *a priori* posited to be uniformly distributed). But the quantum mechanical transformation theory imposes *another* answer, directly *opposed* to this one. Namely, it asserts that the probability measure on the universe of elementary (meta)events $(\pi_k(\psi, \Omega_1), \pi(\psi, \Omega_2))$ is a Dirac dispersion-free measure that associates the probability 1 to the unique joint event $(\pi_k(\psi, \Omega_1), \pi(\psi, \Omega_2))$, where $\pi(\psi, \Omega_2) \in \Sigma_2$ is related with the known measure

$\pi_k(\psi, \Omega_1) \in \Sigma_1$ *according to the set of equations* $\pi(\psi, \omega_{2j}) = |\sum_m \alpha_{jm} c_k(\psi, \omega_{1m})|^2$, $\forall j \in J$, $\forall m \in M$ (J, M are index sets), while the probability of any other one of the considered joint events $(\pi_k(\psi, \Omega_1), \pi(\psi, \Omega_2))$ is posited 0. Which means maximal, deterministic "dependence."

Obviously, the "deterministic probabilistic metadependence" defined above transcends the Kolmogorovian concept of probabilistic dependence. Probabilistic dependence in Kolmogorov's sense does not concern the *meta* concept of whole probability measures, it concerns *individually* and on the *first* level of probabilistic conceptualization two distinct events from the algebra from a probability space. As to correlation (functional relation) between two whole measures from two distinct spaces, when it is asserted in Kolmogorov's sense it involves that these two spaces can be imbedded into a unique product space where the universe of elementary events is produced—physically—by *one* random phenomenon, and where the elementary events from the primitive spaces reappear now *in the algebra* as (in general) dependent events, in Kolmogorov's sense. The quantum mechanical concept of "deterministic probabilistic metadependences" repels these characters.

3.2. The "Potential-Actualization-Actualized" Character of a Probability Tree

Any criterion of mutual consistency of a set of probability measures (Accardi, 1983; Gudder and Zanghi, 1984; Pitowski, 1986; Beltrametti and Maczynski, 1991) certainly reflects some factual unity inside the involved set of random phenomena. The relations (4)—insofar as they are required by the quantum mechanical transformation law (2)—can be regarded as a quantum mechanical condition of mutual consistency of the set of probability measures from a probability tree. Now, the mathematical nucleus of the relations (4) entailed by (2) consists of the unique state vector $|\psi\rangle$ concerned by that tree, that labels the unique result of a given operation P_ψ of state preparation:

- *The quantum mechanical "deterministic probabilistic metadependences" (4) between probability measures from distinct branches of a given probability tree reflect the factual oneness of the studied state from the common trunk of that tree.*

Indeed the state with state vector $|\psi\rangle$, repeatedly reproduced by the reiterations of the operation P_ψ of state preparation, somehow "is" there each time that one operation P_ψ has been achieved: in a certain purely physical-operational sense this state has been created, *defined*, it has been extracted out of the continuum of "reality" and endowed with *physical specificities* imprinted upon it by the operation P_ψ. But these specificities—if we take the liberty to posit their ontological being—lie nevertheless outside the realm of the observable. So from the point of view of *knowledge* they hold the role of merely a certain just posited monolith of indefinitely many still nonrealized sets of *potentialities* of outcomes of only possible but indefinitely many future processes of observation that can be performed on what is labeled $|\psi\rangle$.

- *The operation of state preparation P_ψ acts as a* noncognitive, *purely* physical *definition of an* infinite *set, but a set of mere potentialities.*

With respect to the Frege–Cantor theory of infinite sets as well as with respect to all the logical approaches proposed so far, this is an essential innovation (Mugur-Schächter, 1992*c*, pp. 254–260).

The ontological content conceived for the indefinitely many infinite sets of still nonrealized potentialities physically defined by the operation of state preparation P_ψ is nonremovably *relative* to corresponding indefinitely many conceivable future processes of observation. This relativity generates, for the monolith of potentialities labeled $|\psi\rangle$, classes $M_X, M_Y, \ldots$ of mutually incompatible processes of *actualization* of this or that particular set $\{V_{Xj}\}$, $\{V_{Yk}\}, \ldots$ of *actualized* observable manifestations of *interaction* of the studied entity (state), with this or that sort of macroscopic device $D_X, D_Y, \ldots$. On these various sets of actualized manifestations of interaction are then founded the various possible *actualized* observable branch-probability spaces from the considered tree.

- *The probability tree of a state with state vector $|\psi\rangle$ is a unity endowed with a "potential-actualization-actualized character" ("potential" by what is labeled $|\psi\rangle$; "actualization" by the measurement evolutions M_Ω; "actualized" by the registered eigenvalues $\omega_j = f_\Omega(V_{\Omega j})$.*

In other terms, and this is in striking contrast with the role assigned to the random phenomenon in Kolmogorov's approach:

- *The random phenomenon from any given factual quantum mechanical probability chain* (1″) *is posited to* factually create *the elementary events from the chain.*

The elementary quantum mechanical projectors onto the one-dimensional subspaces from a basis of the Hilbert space of a microsystem, are in fact "factual generators." The purely geometric character assigned to them in the calculus is factitious and deprives them of the time and change that they involve. The formalism of quantum mechanics occults the durations of the individual measurement evolutions that produce the factual quantum mechanical elementary events from the factual chains (1″). Nevertheless, their hidden temporal dimension imprints a nonremovable mark on the formalism, in particular on its factually significant logical features (Mugur-Schächter, 1992*b*, pp. 974–983). Once this is clear, one understands intuitively why formal proofs of incompatibility between the quantum mechanical formalism and hypotheses of "objectification"—weak or strong (Busch and Mittelstaedt, 1991; Busch *et al.*, 1992)—are certainly correct in their conclusion, no matter how these proofs are constructed.

Now, while the "deterministic probabilistic metadependences" (4) between the probability measures from two distinct branches of one tree, regarded as wholes, reflect the oneness of the studied state with state vector $|\psi\rangle$ from the common trunk of that tree, we assert that:

(a) The absence, *beneath* the global functional relation (4), of joint or conditional probabilities relating events (elementary or not) from two distinct branches of a tree reflects the fact that a quantum mechanical random phenomenon is conceived to create ontologically the corresponding elementary events.

(b) The interference of probabilities *in the sense* (2) *of the quantum mechanical transformation theory* reflects the fact that the elementary events V_{Xj} and V_{Yk} that determine, respectively, the eigenvalues $\omega_{Xj} = f_{Xj}(V_{Xj})$ and $\omega_{Yk} = f_Y(V_{Yk})$ of two noncommuting observables Ω_X and Ω_Y tied to two incompatible classes of measurement evolutions M_X and M_Y, cannot be actualized simultaneously for *one* replica of the studied state.

Preliminarily, these contentions can be upheld by a comparison with Kolmogorov's representation. Afterward, with the information theory, we shall obtain a deeper insight.

3.3. Kolmogorov Transformation Theory

Kolmogorov's representation presupposes, implicitly but quite essentially, that the elementary events from a universe U and the events from the algebra τ considered on U are entities (objects, states) with properties which—ontologically—*all* preexist *actualized* to the considered random phenomenon. The paradigm is extraction from an urn. The random phenomenon which brings forth a probability space $[U, \tau, \pi]$ is assumed to describe exclusively the emergence of cognitive connections between the observer's consciousness and these ontologically preexisting actualized properties. Let us examine the formal consequences of this posit.

Imagine an urn with N objects in it, each one of which possesses actual qualifications of two types, a type of qualification Aj of some "nature" A, realized via the "values" j ($j = 1, 2, \ldots, n$) that the nature A assumes, and a type of qualification Bk of a "nature" B, realized via the "values" k

($k = 1, 2, \ldots, m$) assumed by the nature B (for instance: A = form $= f$ and j = cubic, spherical, ..., pyramidal; B = color $= c$ and k = red, white, ..., dark). In *these* conditions the classical theory of probabilities introduces the expressions

$$p(Aj) = \sum_k p(Aj, Bk) = \sum_k p(Aj/Bk)p(Bk) = \sum_k \alpha_{jk} p(Bk) \qquad (5)$$

where $p(Aj, Bk)$ is the joint probability to extract an object with qualification j of nature A and qualification k of nature B, and $p(Aj/Bk) = \alpha_{jk}$ is the conditional probability to extract an object with qualification k of nature B, given that its qualification of nature A is j. In (5), just as in the quantum mechanical "transformation relation" (2), $|c(\psi, \omega_{2j})|^2 = |\sum_k \alpha_{kj} c(\psi, \omega_{1k})|^2$, the probability of emergence of a qualification j from a given class A is expressed as a function of the probabilities of emergence of all the possible qualifications k from another class B. In this sense (5) also is a "transformation law," like (2). But in (5) the probabilities $p(Bk)$ do not "interfere" inside the second part of the equation, i.e., the probability $p(Aj)$ is a linear combination of the probabilities $p(Bk)$. This formal character translates directly the assumption of the *existence* of definite joint probabilities $p(Aj, Bk)$ for the events (Aj, Bk), so of definite conditional probabilities $p(Aj/Bk)$ [since these, in (5), are themselves the coefficients α_{jk} of linear combination]. In its turn, the assumption of existence of joint and conditional probabilities is entailed by the more basic assumption of actualized ontological preexistence, for each object from the urn, of an A-and-B qualification: It is in consequence of this last assumption that a random phenomenon where the identically reproducible procedure is reducible to the paradigm of extraction from the urn can produce a universe of elementary events consisting of joint qualifications (Aj, Bk). Then each qualification (Aj) or (Bk)—separately—labels an event from the total algebra on this universe of joint elementary qualifications, and *all* the considered probability assignations $p(Aj)$, $p(Bk)$, $p(Aj, Bk)$ are embedded into a unique probability space, thus forming a "classical polytop" (Pitowski, 1989) or a "classical correlation sequence" (Beltrametti and Maczynski, 1991). In short, the semantic assumption of actualized ontological preexistence of all the considered qualifications generates all the syntactical characteristics of the Kolmogorov concept of probability, in particular, the linear character of the Kolmogorov transformation law (5).

Notice that the same formal effect (5) can be obtained also via the less restrictive semantic assumptions of either (a) the possibility of *simultaneous actualization* of ontological properties of the studied entity producing all the considered qualifications (elementary events), or (b) the possibility of actualization of only one ontological property of the studied entity which in its turn produces simultaneously all the considered qualifications, as happens inside one branch of a quantum mechanical probability tree.

3.4. Incompatibility with Quantum Mechanics of the Kolmogorov Transformation Law

The formalism of quantum mechanics is incompatible with a general acceptance of any one of the semantic assumptions able to entail (5). The various sets $\{V_{Xj}\}$, $\{V_{Yk}\}, \ldots$ of mutually incompatible qualifications produced by the random phenomena stemming from a single operation of state preparation are *all* assumed quite essentially *not* to preexist ontologically actualized: if they were not, it would not be possible to associate them all with a single operation of state preparation. So two such sets $\{V_{Xj}\}$ and $\{V_{Yk}\}$ can only belong to two *different* random phenomena involving two incompatible classes of individual measurement evolutions M_X and M_Y, respectively. So no universe of elementary events consisting of simultaneous (joint) qualifications (V_{Xj}, V_{Yk}) can ever be both *factually* produced for the *same single* replica of the studied entity. Then each one of the sets $\{V_{Xj}\}$, $\{V_{Yk}\}, \ldots$ —in the role of a universe of *elementary* events—can only found its *own* factual probability space, structurally different from all the other ones. No unique probability space is factually constructible where it is possible, as for Kolmogorov probabilities, to locate definite *individual* conditional probabilistic connections $p[(V_{Xj})/V_{Yk}]$ between qualifications from the two sets $\{V_{Xj}\}$ and $\{V_{Yk}\}$, belonging all to the same "classical polytop." In *such* conditions a linear transformation law of the type (5) *would be devoid of factual counterpart.*

And indeed—faithful to operationality—the quantum mechanical transformation theory asserts instead the transformation law (2), $|c(\psi, \omega_{Yk})|^2 = |\sum_j \alpha_{kj} c(\psi, \omega_{Xj})|^2$, where in the second part of the equality, besides a linear superposition of the probabilities $|c(\psi, \omega_{Xj})|^2$, there appear also other, "interference" terms: This is a formal signature of the basic semantic difference between Kolmogorov probabilities and quantum mechanical probabilities.

4. QUANTUM MECHANICS VERSUS INFORMATION THEORY

Formally it is clear how the quantum mechanical bra-ket algebra produces the transformation algorithm (2). Also, it is quite remarkable that it does produce it. Yet, the algorithms of quantum mechanics offer no clue whatever permitting one to *understand* the physical and conceptual implications of the quantum mechanical transformation theory; they suggest no model concerning the real processes that bring forth the results asserted by them.

On the other hand, consider the theory of information. This theory —quite basically—involves probability laws. However, the *deep* relation, within this theory, between numerical probabilistic estimations and the other representations remains obscure.

We shall now show a far-reaching fact: namely, that in its essence the informational concept of probability is the same as the quantum mechanical one, so fundamentally distinct from the Kolmogorov concept of probability. Guided by the recognition of this semantic unity, we shall be able, despite radical differences between the quantum mechanical system of representation and the informational one, to sketch out an informational transformation theory which is clearly comparable with the quantum mechanical transformation theory. This amounts to the construction of an *intelligible model* for the physical processes involved. The way in which this model emerges, the nature of the encountered resistances, and the content of the result, suggest the possibility to synthesize a new formalism where a generalized bra-ket algebra on a vector space is explicitly related to the main elements of the general informational representations of input and output sources connected by channels. Such a formalism might be able to represent mathematically the emergence and circulation of patterns of any kind.

4.1. Information Trees

Consider an information source $S = \{(Ai, p(Ai)), i = 1, 2, \ldots, n\}$ with zero memory, which emits an input alphabet $A = \{Ai\}$ with input probability law $p(Ai)$ on it. Consider also an information channel C, which, when it is associated with the input source S, yields an output alphabet $B = \{Bk\}$ with an output probability law $p(Bk)$ on it, $k = 1, 2, \ldots, r$. Together, the input source S and the channel C form an information system $I(S, C)$. In the most general case any input sign Ai can, with a certain conditional probability $p(Bk/Ai)$, produce any output sign Bk (in particular, $p(Bk/Ai)$ can be 0 for this or that pair (Ai, Bk)). Inside $I(S, C)$, the channel C is defined—*relative to the information source* S—by a channel-matrix $\mathbf{M}(C/S)$ of which the elements m_{ik} are the conditional probabilities $p(Bk/Ai)$ for an output Bk, given a definite input Ai:

$$\mathbf{M}(C/S) = [m_{ik}] = [p(Bk/Ai)]$$

where the possible inputs Ai are displayed in row and the possible outputs Bk are displayed in column. The total probability of an output Bk is calculated as

$$p(Bk) = \sum_i p(Ai)p(Bk/Ai)$$

If in particular the matrix $\mathbf{M}(C/S)$ is such that each input sign can produce at most one output (each row from $\mathbf{M}(C/S)$ contains only one nonzero element, which is 1), the information system $I(S, C)$ is called "deterministic with noise," because it ensures certain prediction but only probabilistic retrodiction.

If $\mathbf{M}(C/S)$ is such that one given output can be produced by only one input (each column contains only one nonzero element), the information system $I(S, C)$ is qualified as "nondeterministic without noise," because it ensures certain retrodiction but only probabilistic prediction.

The central point in this context is that an input alphabet $\{(Ai, p(Ai)), i = 1, 2, \ldots, n\}$ emitted by the input source S from an information system $I(S, C)$ denotes an ontological initial content which in general is *transformed* by passage through the channel C from that system. The channel C *factually creates* the observed output alphabet $\{(Bk, p(Bk)), k = 1, 2, \ldots, r\}$. So, with respect to the channel C and the observed output alphabet, the input source $S = \{(Ai, p(Ai)), \ i = 1, 2, \ldots, n\}$ acts as a mere potentiality. And with respect to another channel $C' \neq C$, this *same* source S acts as a *different* potentiality, in the sense that it entails another observable output alphabet, $\{(B'k, p'(B'k))$. So *the input source* $S = \{(Ai, p(Ai)), i = 1, 2, \ldots, n\}$ *can also be regarded as a set of indefinitely many potentialities, relative to the indefinitely many channels to which it can be connected.* Between an input Ai and an output Bk there is process, there is "relative time", time populated by change relative to the acting pair (S, C). The conditional probabilities $p(Bk/Ai)$ from the matrix of the acting channel C with respect to the acting source S are *Bayes* conditional probabilities, *not* Kolmogorov conditional probabilities (Jaynes, 1979). But this reproduces the very essence of the concept of a quantum mechanical probability tree. This concept, then, quite fundamentally, must be transposable in informational terms. Let us perform the transposition.

Consider a quantum mechanical probability tree. The operation P_ψ of state preparation that generates the tree can be regarded as an "information source" without memory. According to orthodox quantum mechanics, such an information source emits only *one* input sign, namely the state represented by the state vector $|\psi\rangle$. Furthermore, each type of process of measurement evolution $M_X, M_Y, \ldots$ from a branch of the probability tree can be regarded as a particular sort of information channel; let us call it a "quantum measurement information channel" $C_X, C_Y, \ldots,$ producing, on a corresponding output device $D_X, D_Y, \ldots,$ an output alphabet $\{V_{Xj}\}, [V_{Yk}\}, \ldots,$ respectively. So—according to *orthodox* quantum mechanics—each branch of a probability tree acts as *a nondeterministic information system* $I(P_\psi, C_X), I(P_\psi, C_Y), \ldots$ *without noise* corresponding to a set of observables $\{\Omega^h_X, \ h = 1, 2, \ldots, l\}$ all tied to the same class M_X of individual measurement evolutions, or, respectively, to a set of observables $\{\Omega^g_Y, \ g = 1, 2, \ldots, s\}$ all tied to another same class M_Y of individual measurement evolutions, etc. For the information system $I(P_\psi, C_X)$, for instance, the channel matrix and the output probability law are

$$\mathbf{M}(C_X/P_\psi) = [m_{\psi j}] = [p(V_{Xj}/\psi)]$$
$$p(V_{Xj}) = p(\psi)p(V_{Xj}/\psi) = p(V_{Xj}/\psi) \tag{6}$$

with $p(\psi) = 1$ (Mugur-Schächter, 1991, pp. 1402–1405) and where $p(V_{Xj}/\psi) = p(P_\psi, M_X, V_{Xj})$ are the quantum mechanical *factual* probability densities from (1″) which, via the functional relations $\omega^h_{Xj} = f^h_X(V_{Xj})$, generate all the quantum mechanical eigenvalues ω^h_{Xj} and the corresponding predictional probability densities $\pi(\psi, \omega^h_{Xj}) = |c(\psi, \omega^h_{Xj})|^2$ for all the observables Ω^h_X. Because the input alphabet emitted by P_ψ contains only one input sign $|\psi\rangle$ from (6), $\mathbf{M}(C_X/P_\psi)$ from (6) is a *column* matrix of *factually* observed probabilities $p(V_{Xj}/\psi)$ that, by the expansion coefficients $c(\psi, \omega^h_{Xj})$, generates all the column-ket matrixes representing the state vector $|\psi\rangle$ with respect to the basis of common eigenkets of all the observables Ω^h_X tied to the eigenvalues ω^h_{Xj} of this or that observable Ω^h_X, $h = 1, 2, \ldots, l$:

- *A quantum mechanical probability tree $\mathbf{T}(P_\psi)$ can be regarded also as an "information tree" $I\mathbf{T}(P_\psi)$, i.e., as a branching structure of information systems obtained when one given input source is combined with all the mutually incompatible "quantum measurement channels" connectable to that source.*

This shows that—in its semantic essence—the informational concept of probability can be identified with the quantum mechanical one.

4.2. An Informational Transformation Theory?

In such conditions one expects furthermore an informational–Bayes transformation theory of the same type, in essence, as the quantum mechanical one. So we now ask: What, *expressed in the language of the theory of information*, is the relation between two output probability laws, regarded as wholes, corresponding to two different branches from the information tree $I\mathbf{T}(P_\psi)$ corresponding to a given state vector $|\psi\rangle$? For instance, between the output probability laws produced from $|\psi\rangle$ by two incompatible processes of measurement evolutions $M_X \Leftrightarrow C_X$ and $M_Y \Leftrightarrow C_Y$?

Now, as far as we know, information theory, as it now stands, does not contain an answer to this question. *There is no informational transformation theory* stating a general relation between the output probability laws of incompatible information systems involving the same source.

Moreover, it is not even *possible* to work out such an answer so long as one conserves the hypothesis of only *one* input sign (in our case $|\psi\rangle$) produced by the considered source (in our case P_ψ): with this highly degenerate hypothesis that entails in (6) a *column* matrix $\mathbf{M}(C_X/P_\psi)$, the informational formalism—contrary to the quantum mechanical vector space bra-ket algebra—offers no indications whatever as to how one could advance beyond the *separate* assertion of each one of the mutually incompatible output laws connected with $|\psi\rangle$, in order to elaborate a connection between these. Inside the informational system of representation the situation seems to be blocked.

4.3. Macroscopic–Microscopic Representation of an Information System

However—and this is a noteworthy fact—the obstacle, though it does not dissolve, recedes as soon as one supposes "hidden variables."

Suppose that a quantum mechanical operation of state preparation P_ψ creates, at the *microscopic* level of specification, a physical mode of existing of the studied state labeled by the state vector $|\psi\rangle$ that can be globally characterized by one from a whole set $\Lambda = \{\lambda_i, i = 1, 2, \ldots, n\}$ of n possible different "hidden" input signs, $n > 1, p(\lambda_i)$ being the input probability of a given $\lambda_i \in \Lambda$. Now, for reasons of descriptional homogeneity, a microscopic input state represented by an input sign λ_i can only be conceived to combine directly with an equally microscopic channel state. So suppose that in each *one* realization of an elementary quantum mechanical chain experiment P_ψ-M_X-V_{Xj} the C_X-channel situation corresponding to the involved individual measurement evolution M_X, such that in *that* realization it emerges at the microscopic level, can be globally characterized by one from a whole set of different possible "hidden" channel-state signs, respectively $\{\mu_{rX}, r = 1, 2, \ldots, m\}, m > 1$.

Consider now two mutually incompatible measurement evolutions M_X and M_Y. We introduce the following symbols.

- $\{(\lambda_i, \mu_{rX})\}^{\to Xj}$ and $\{(\lambda_i, \mu_{sY})\}^{\to Yk}$, respectively, are the set of all the pairs (λ_i, μ_{rX}) that can contribute to the output V_{Xj}, and the set of all pairs (λ_i, μ_{sY}) that can contribute to the output V_{Yk}. Also, $(\lambda_i, \mu_{rX})^{\to Xj}$ and $(\lambda_i, \mu_{sY})^{\to Yk}$, respectively, are an element of $\{(\lambda_i, \mu_{rX})\}^{\to Xj}$ and an element of $\{(\lambda_i, \mu_{sY})\}^{\to Yk}$.
- $\{\lambda_i\}^{\to Xj}$ and $\{\lambda_i\}^{\to Yk}$, respectively, are the set of all the input signs λ_i that can contribute to the output V_{Xj}, and the set of all the input signs λ_i that can contribute to the output V_{Yk}. Also, $\lambda_i^{\to Xj}$ and $\lambda_i^{\to Yk}$, respectively, are an element from $\{\lambda_i\}^{\to Xj}$ and an element from $\{\lambda_i\}^{\to Yk}$.
- ${}^i\mu_{rX}^{\to j}$ is a channel sign μ_{rX} such that, if associated with λ_i, it generates a pair $(\lambda_i, \mu_{rX})^{\to Xj}$, and, ${}^i\mu_{sY}^{\to k}$ is a channel sign μ_{sY} such that, if associated with λ_i, it generates a pair $(\lambda_i, \mu_{sY})^{\to Yk}$.

The notations containing an arrow are "predispositional" notations, in the sense that they qualify capacities of the input signals λ_i *relative* to this or that possible future observable output V_{Xj}, or V_{Yk}, etc. In agreement with statistical thermodynamics, the defined notations presuppose the condition:

(C_{ST}) For any channel C_X, one given observable macroscopic output V_{Xj} in general can stem from any pair $(\lambda_i, \mu_{rX}) \in \{(\lambda_i, \mu_{rX})\}^{\to Xj}$.

We furthermore admit the following rather unavoidable "deterministic" postulate:

(P_D) A given pair $(\lambda_i, \mu_{rX})^{\to Xj}$ can produce only one corresponding macroscopic output V_{Xj}.

Consider now a predispositional set $\{\lambda_i\}^{\to Xj}$. The information theory admits in general nonrestricted possibilities concerning the transformations from an input signal λ_i to an output sign V_{Xj}:

(C_{IT}) For any channel C_X, each input sign λ_i is able to contribute either to any output sign from the output alphabet $\{V_{Xj}\}$, or to only some of these signs, or to *none*; so a given output sign V_{Xj} can be connected with several or all input signs λ_i.

But the formalism of quantum mechanics implies the following restrictive condition C_{QM}:

(C_{QM}) Each elementary quantum mechanical chain experiment P_ψ-M_X-V_{Xj} from any quantum mechanical random phenomenon does end with *some* factual result V_{Xj}. So each input sign λ_i contributes to at least one output sign V_{Xj}, for any $I(P_\psi, C_X)$.

In consequence of C_{ST} and P_D, with respect to the set $\{(\lambda_i, \mu_{rX}), i = 1, 2, \ldots, n, r = 1, 2, \ldots, m)\}$ of *all* the possible pairs (λ_i, μ_{Xr}) (regarded as a new input alphabet produced by the individual interactions between the source P_ψ and a quantum measurement channel C_X, entirely specified at the microscopic level of description), an information system $I(P_\psi, C_X)$ acquires now the structure of *a deterministic system with noise*, ensuring certain prediction but only probabilistic retrodiction. Compared with the initial direct informational transcription (6) of the *orthodox* quantum mechanical assumptions, which require only one input sign $|\psi\rangle$ and consequently translates into a nondeterministic information system $I(P_\psi, C_X)$ without noise, the situation appears as simply *reversed.*

So, instead of highly degenerate channel matrixes of the form (6), a microscopic–macroscopic characterization yields now the following quantum-measurement-channel matrixes for any two mutually incompatible information systems $I(P_\psi, C_X)$ and $I(P_\psi, C_Y)$ from the information tree $I\mathbf{T}(P_\psi)$ of the operation P_ψ of state preparation:

$$\begin{aligned} \mathbf{M}(C_X/P_\psi) &= [m_{X,ij}] = [p(V_{Xj})/(\lambda_i, \mu_{rX})] = [\delta_{ir,j}] \\ \mathbf{M}(C_Y/P_\psi) &= [m_{Y,ik}] = [p(V_{Yk})/(\lambda_i, \mu_{sY})] = [\delta_{is,k}] \end{aligned} \tag{7}$$

where, by definition, $\delta_{ir,j} = 1$ if the pair of indexes i, r corresponds to a pair $(\lambda_i, \mu_{rX}) \in \{(\lambda_i, \mu_{rX})\}^{\to Xj}$, and otherwise $\delta_{ir,j} = 0$; $\delta_{is,k} = 1$ if the pair of indexes i, s corresponds to a pair $(\lambda_i, \mu_{Ys}) \in \{(\lambda_{ik}, \mu_{sY})\}^{\to Yk}$, and otherwise $\delta_{is,k} = 0$. Furthermore, *the channel situation is independent of the activity of the source*, so we just set $p(\lambda_i, \mu_{rX}) = p(\lambda_i)p(\mu_{rX})$. This, together with the form (7) for the channel matrixes, entails for $p(V_{Xj})$ and $p(V_{Yk})$ the following successive expressions

$$
\begin{aligned}
p(V_{Xj}) &= \sum_i \sum_r p(\lambda_i, \mu_{rX}) p(V_{Xj}/(\lambda_i, \mu_{rX}) \\
&= \sum_i \sum_r p(\lambda_i, \mu_{rX})^{\to Xj} = \sum_i p(\lambda_i) \sum_r p({}^i\mu_{rX}^{\to j})\delta_{ir,j} \\
p(V_{Yk}) &= \sum_i \sum_s p(\lambda_i, \mu_{sY}) p(V_{Yk}/(\lambda_i, \mu_{sY}) \\
&= \sum_i \sum_r (\lambda_i, \mu_{sY})^{\to Yk} = \sum_i p(\lambda_i) \sum_s p({}^i\mu_{sY}^{\to k})\delta_{is,k}
\end{aligned}
\tag{8}
$$

The representation (7), (8) defines—for the particular case of the information systems $I(P_\psi, C_X)$, $I(P_\psi, C_Y)$, etc., from a quantum mechanical information tree $I\mathbf{T}(P_\psi)$—a two-level *extension* of the customary one-level informational representations: the macroscopic (V_{Xj}, V_{Yk}) and the microscopic $(\lambda_i, \mu_{rX}, \mu_{sY})$ levels of any information system $I(C_X/P_\psi)$ from $I\mathbf{T}(P_\psi)$ are now both explicitly represented and connected. This entails already a nontrivial consequence:

- *The representation (7), (8) yields, for the quantum measurement theory from any attempt at a hidden variables interpretation of quantum mechanics, a general informational framework, where now also any individual quantum mechanical chain experiment is represented, namely by a corresponding sequence P_ψ-$(\lambda_i, \mu_{r\Omega})$-$V_{\Omega j}$ of a macroscopic operation P_ψ, a nonobservable microscopic pair $(\lambda_i, \mu_{r\Omega})$ and a macroscopic observable output $V_{\Omega j}$.*

4.4. On an Informational Transformation Law

In contradistinction to the initial one-level representation (6), does the two-level representation (7), (8) permit us finally to express in informational language a transformation law?

The answer will be very instructive: It will appear that, starting from the two-level representation (7), (8) and taking into account, as a guide, the essential characteristics of the quantum mechanical transformation law (2), it *is* possible to build at least one informational expression of a transformation law. This expression can be obtained in a form which—in a certain quite fundamental sense that stems directly from the basic semantic identity between the quantum mechanical and the informational concepts of probability—is the "same" as the quantum mechanical form (2). But such an informational expression of a transformation law can be drawn into existence only by struggling against strong descriptional resistances; correlatively, it is *not effectively computable*, in contradistinction to the quantum mechanical law (2). Quantum mechanics represents its particular sort of information systems in a way which is *radically* different from the informational one, far more performable from a computational standpoint, and possibly, conceptually innovating. The contrast will suggest possibilities of a new, synergetic representation.

Preliminarily, let us notice that—from the outset—an informational transformation law of the Kolmogorov type (5) is excluded, since in an information tree $I\mathbf{T}(P_\psi)$, just as in a quantum mechanical tree, the output signs are factually created by passage of the input signals through the acting channel, so one cannot define *factually significant* conditional probabilities relating directly an output V_{Xj} produced by a channel C_X with outputs V_{Yk} produced by another channel C_Y that is incompatible with C_X.

Consider now the quantum mechanical transformation law (2)

$$\begin{aligned}\pi((\psi, \omega^h_{Xj}) &= |c(\psi, \omega^h_{Xj})|^2 \\ &= \left|\sum_k \alpha_{jk} c(\psi, \omega^{h'}_{Yk})\right|^2 \\ &= \sum_k |\alpha_{jk}|^2 |c(\psi, \omega^{h'}_{Yk})|^2 + [\text{"interference" terms}] \\ &= \sum_k |\alpha_{jk}|^2 \pi((\psi, \omega^{h'}_{Yk}) + [\text{"interference" terms}]\end{aligned}$$

This law equates the probability density $\pi((\psi, \omega^h_{Xj})$ of a eigenvalue ω^h_{Xj} of an observable Ω^h_X tied to measurement evolutions M_X, with a linear combination $\sum_k |\alpha_{jk}|^2 \pi((\psi, \omega^{h'}_{Yk})$ of the probabilities $\pi((\psi, \omega^{h'}_{Yk})$ of all the eigenvalues $\omega^{h'}_{Yk}$ of another observable Ω^h_Y tied to measurement evolutions M_Y incompatible with the evolutions M_X, *plus* ["interference" terms]. Examine the involved coefficients, $\alpha_{jk} = \langle v_k | u_j \rangle$. These are the elements of the transformation matrix S_{XY} from the basis $\{|u_j\rangle\}$ of common eigenvectors $|u_j\rangle$ of all the observables Ω^h_X tied to the measurement evolution M_X from the channel C_X, to the basis $\{|v_k\rangle\}$ of common eigenvectors $|v_k\rangle$ of all the observables $\Omega^{h'}_Y$ tied to the measurement evolutions M_Y from the channel C_Y. By means of the passage from the eigenvector $|u_j\rangle$ to the eigenvector $|v_k\rangle$, a number $\alpha_{jk} = \langle v_k | u_j \rangle$ characterizes the passage from the factual output V_{Xj} corresponding to $|u_j\rangle$ to the factual output V_{Yk} corresponding to $|v_k\rangle$ [the "degenerate" relations between eigenvalues being accounted for with the help of the involved connective functions $\omega^h_{Xj} = f^h_X(V_{Xj})$, $\omega^{h'}_{Yk} = f^h_Y(V_{Yk})$, etc.]. So each coefficient α_{jk} has a value depending on the involved pair of factual outputs V_{Xj} and V_{Yk}: $\alpha_{jk} = \alpha_{jk}(V_{Xj}, V_{Yk})$. Furthermore, notice now that each coefficient α_{jk} is: (a) quite *independent* of the involved concept of source (the operation of state preparation P_ψ and its input sign $|\psi\rangle$), and (b) free of any *probabilistic* connotation. But a basis $\{|u_j\rangle\}$ of common eigenvectors $|u_j\rangle$ of all the observables Ω^h_X tied to the quantum measurement channel C_X and the corresponding set $\{V_{Xj}\}$ of factual outputs constitute together the quantum mechanical characterization of C_X:

- *The formalism of quantum mechanics characterizes a "quantum measurement channel" C_X* independently *of any information source, and via the* nonprobabilistic *concept of a family $\{|u_j\rangle\}$ of eigenvectors $|u_j\rangle$, each one of which represents formally—by a function—a type of "signal" that remains invariant by passage through C_X and produces a factual observable output sign V_{Xj}.*

The descriptional strategy that generates the quantum mechanical transformation law (2) is *divide ut impera*: In the formal language of orthodox quantum mechanics the umbilical cord between input sources and channels is neatly cut. The quantum measurement channels C_X are described "intrinsically" and in a nonprobabilistic way by their invariant input signals $|u_j\rangle$, formally determined by the observables Ω^h_X via the equation $\Omega^h_X|u_j\rangle = \omega^h_{Xj}|u_j\rangle$. The input probabilities for an information system $I(P_\psi, C_X)$ from an information tree $I\mathbf{T}(P_\psi)$ simply are not defined [or are trivially defined as $\pi(\psi) = 1$; see (6)]; they are skipped. The accent is put *exclusively* on the operational-observational obtaining of the output signs V_{Xj} entailed by the invariant input signals of C_X, the eigenvectors $|u_j\rangle$ from the common basis of all the observables Ω^h_X. The probabilities $p(V_{Xj})$ of these output signs are calculated with the help of the association of (a) the principle of spectral decomposability $|\psi\rangle = \sum_j \langle u_j|\psi\rangle|u_j\rangle$ of the mathematical representation of the unique input $|\psi\rangle$ emitted by P_ψ and (b) the "predictional" probability postulate $\pi((\psi, \omega^h_{Xj}) = |\langle u_j|\psi\rangle|^2$. This permits one to represent *nonlinear effects of interaction* between any pair of two distinct input sources $P_{\psi1}, P_{\psi2}$ that are both involved in an "interaction source" $P_{\psi12\ldots n} = G(P_{\psi1}, P_{\psi2}, \ldots, P_{\psi n})$, notwithstanding the use of a fundamentally linear, vector-space representation for the input signals $|\psi\rangle$ emitted by the considered input sources P_ψ and for the "eigeninput" signals $|u_j\rangle$ of a "quantum measurement channel" C_X.

The descriptional strategy of information theory is very different. Within the informational language, the concept of an observable, with eigenvectors and eigenvalues, is *not defined* (nor is a channel regarded as necessarily being a "measurement"). The successive phases of the process of "transmission of information"—emission of an input signal Ai, passage through the channel C from the considered information system $I(S/C)$, observation of an output signal Bk created out of Ai by the passage of Ai through C—are *all* explicitly represented, according to a very intuitive view. The representation of all the phases is *probabilistic*, and the umbilical cord between channel representation and the acting input source is *not* cut:

- *The formalism of information theory characterizes a channel by a matrix* $\mathbf{M}(C/S)$ *of which the elements* $m_{ik} = p(Bk/Ai)$ *are conditional probabilities of an output sign* Bk*, an input sign* Ai *from* S *being given, so in a way that is essentially dependent on the considered input source* S*, and essentially probabilistic.*

Such a representation is not adapted for expressing a transformation law. This is why there is no informational transformation theory.

In such conditions it seems fit, in order to investigate on the possibility, in *principle*, of an informational transformation law, and on its nature when *compared* with the quantum mechanical one, to utilize the quantum mechanical expression (2) as a close guide.

So, to begin with, let us seek, inside information theory, a linear combination $\sum_k \alpha'_{jk} p(V_{Yk})$ of all the probabilities $p(V_{Yk})$ where the coefficients α'_{jk} are required as constants of which the values, as in the case of the quantum mechanical transformation coefficients $\alpha_{jk} = \langle v_k | u_j \rangle = \alpha_{jk}(V_{Xj}, V_{Yk})$, somehow characterize the passages from a factual output V_{Xj} possible by the use of the channel C_X, to a factual output V_{Yk} possible when the channel C_Y is at work: $\alpha'_{jk} = \alpha'_{jk}(V_{Xj}, V_{Yk})$. Now, with the assumed representation (7), (8), the only available pools of descriptional elements for building the sought numbers $\alpha'_{jk}(V_{Xj}, V_{Yk})$ are either the conditional probabilities $p(V_{Xj})/(\lambda_i, \mu_{rX})$ and $p(V_{Yk})/(\lambda_i, \mu_{sY})$ from the channel matrixes (7) of C_X and C_Y, or directly the expressions (8) of the output probabilities $p(V_{Xj})$ and $p(V_{Yk})$. The conditional probabilities $p(V_{Xj})/(\lambda_i, \mu_{rX})$ and $p(V_{Yk})/(\lambda_i, \mu_{sY})$ from (7) are all 1 or 0, which is an obstacle in the way of a definition with their help of numbers $\alpha'_{jk}(V_{Xj}, V_{Yk})$ that shall characterize the pair of outputs V_{Xj}, V_{Yk}. So let us examine the expressions (8) of the probabilities $p(V_{Xj})$ and $p(V_{Yk})$. In these, though each probability $p({}^i\mu_{rX}^{\to j})$ or $p({}^i\mu_{sY}^{\to k})$ is also connected with a corresponding input sign λ_i, nevertheless the factors $\sum_r p({}^i\mu_{rX}^{\to j})\delta_{ir,j}$ and $\sum_s p({}^i\mu_{sY}^{\to k})\delta_{is,k}$ do express *channel* features specifically tied to, respectively, V_{Xj} and V_{Yk}. By the use of these factors, however, we can only obtain numbers depending on output indexes j, k, *and* on an input index i. So let us indicate such numbers by the modified symbol ${}^i\alpha'_{jk}$.

The most straightforward definition of a coefficient ${}^i\alpha'_{jk}$ as a number depending on (V_{Xj}, V_{Yk}) is the ratio

$$ {}^i\alpha'_{jk} = \left[\sum_r p({}^i\mu_{rX}^{\to j})\delta_{ir,j} \Big/ \sum_s p({}^i\mu_{sY}^{\to k})\delta_{is,k} \right] \tag{9} $$

where $p({}^i\mu_{rX}^{\to j}) = p(\mu_{rX})$, $p({}^i\mu_{sY}^{\to k}) = p(\mu_s Y)$; ${}^i\alpha'_{jk}$ is posited not to be defined when $\delta_{is,k}$ is 0 for all the indexes s (i.e., if the involved λ_i does not contribute to the output V_{Yk}, which is permitted by the assumed general conditions C_{ST}, C_{IT}, C_{QM}). The ratio (9) can be regarded as an estimation of the "j-efficiency of the channel C_X" relative to the "k-efficiency of the channel C_Y" and with respect to the input sign λ_i.

With the definition (9) and with (8) we can now form a linear combination $\sum_k \alpha'_{jk} p(V_{Yk})$ to be compared with the quantum mechanical one, $\sum_k |\alpha_{jk}|^2 \pi((\psi, \omega_{kY}^{h'})$ from (2). We obtain

$$ \sum_k {}^i\alpha'_{jk} p(V_{Yk}) = \sum_i p(\lambda_i) \sum_k \left[\sum_r p({}^i\mu_{rX}^{\to j})\delta_{ir,j} \Big/ \sum_s p({}^i\mu_{sY}^{\to k})\delta_{is,k} \right] \sum_s p({}^i\mu_{sY}^{\to k})\delta_{is,k} \tag{10} $$

Now, for $k \neq k'$, in general

$$ \sum_s p({}^i\mu_{sY}^{\to k})\delta_{is,k} \neq \sum_s p({}^i\mu_{sY}^{\to k})\delta_{is,k'} $$

So in (10) *each* index k yields, by cancellation of the corresponding $\sum_s p({}^i\mu_{sY}^{\to k})\delta_{is,k}$, its *own* term $\sum_r p({}^i\mu_{rX}^{\to j})\delta_{ir,j}$. Hence (10) contains each term $\sum_r p({}^i\mu_{rX}^{\to j})\delta_{ir,j}$ the number of times $n(i, Y)$ that the corresponding input sign λ_i contributes to *some* output V_{Yk} from the alphabet $\{V_{Yk}\}$:

$$ \sum_k {}^i\alpha'_{jk} p(V_{Yk}) = \sum_i p(\lambda_i) \left[n(i, Y) \sum_r p({}^i\mu_{rX}^{\to j})\delta_{ir,j} \right] \tag{10'} $$

[according to C_{QM}, each λ_i contributes to at least one output V_{Yk}, which excludes $n(i, Y) = 0$, while the upper limit for $n(i, Y)$ is set by the cardinal of $\{V_{Yk}\}$]. But the sum

$$ \sum_i p(\lambda_i)[n(i, Y) \sum_r p({}^i\mu_{rX}^{\to j})\delta_{ir,j}] $$

from (10′) is different, in general, from the sum

$$ \sum_i p(\lambda_i) \sum_r p({}^i\mu_{rX}^{\to j})\delta_{ir,j} $$

from the expression (8) of $p(V_{Xj})$. So, as in quantum mechanics, we have in general

$$ p(V_{Xj}) \neq \sum_k {}^i\alpha'_{jk} p(V_{Yk}) \tag{11} $$

The inequality (11) can be understood intuitively from the "predispositional" properties of the input signs λ_i: The possibilities C_{IT} entail that, in general, the following relations hold for the predispositional input sets:

$$\{\lambda_i\}^{\to Yk} \cap \{\lambda_i\}^{\to Yk'} \neq \varnothing \qquad \text{for} \quad k \neq k' \tag{12}$$

$$\bigcup_k \{\lambda_i\}^{\to Yk} \neq \{\lambda_i\}^{\to Xj} \tag{13}$$

(while the condition C_{QM} only requires $\{\lambda_i\}^{\to Xj} = \bigcup_k [\{\lambda_i\}^{\to Xj} \cap \{\lambda_i\}^{\to Yk}]$). According to (12), each input signal λ_i can be involved in several distinct predispositional events $\{\lambda_i^{\to Yk}\}$, corresponding to different indexes k; *the predispositional events* $\{\lambda_i^{\to Yk}\}$ *interfere in general.* Furthermore, according to (13), the union $\bigcup_k \{\lambda_i\}^{\to Yk}$ of all the Y-predispositional sets $\{\lambda_i\}^{\to Yk}$ might not *exhaust* the Xj-predispositional set $\{\lambda_i\}^{\to Xj}$, or might *exceed* it. So, if in the expression of the probability $p(V_{Xj})$ (j fixed) sought as a function of all the $p(V_{Yk})$ we begin by writing down the linear combination $\sum_k {}^i\alpha'_{jk} p(V_{Yk})$ which, with the choice (9), for any given λ_i, generates a term $p(\lambda_i) \sum_r p({}^i\mu_{rX}^{\to j}) \delta_{ir,j}$ for each *Yk separately*, and then add all these terms, the possible interferences expressed by (12) and the possible inequality (13) are not taken into account according to the law of total probabilities. Therefore in general we shall afterward have to add—algebraically—other terms in order to compensate for the effect on the probabilities of the "interferences" (12) and the nonequality (7).

So, with the choice (9) for the coefficients ${}^i\alpha'_{kj}(V_{Xj}, V_{Yk})$, we finally obtain for the transformation law an informational expression of the form

$$p(V_{Xj}) = \sum_k {}^i\alpha'_{jk} p(V_{Yk}) + [\text{other terms}] \tag{2'}$$

Obviously (2′) is of the same general type as the quantum mechanical transformation law (2). This is a formal similitude which, *across* the deep differences between the quantum mechanical and the informational strategies for representing a quantum measurement channel, stems directly from the semantic identity between the quantum mechanical and the informational concepts of probability. Indeed, the informational relation (2′) appears *explicitly* as a consequence of the fact that the output signs V_{Yk} are assumed to be factually created out of the input signals by passage of these through the "measurement channel" at work, which entails for the input signs λ_i an only "predispositional" role.

But, as announced, in the informational expression (2′) [other terms] is *not effectively computable*, while in the quantum mechanical expression (2) ["interference" terms] is computable. This draws attention to the specific capacities of the quantum mechanical system of representation of information. From the standpoint of information theory, the quantum mechanical representation amounts to just *a formal scenario* according to which each one realization of an operation of state preparation P_ψ is regarded as producing the *whole* alphabet of *input* pairs $\{(|u_j\rangle, V_{Xj})\}$ distributed with probability density $p(|u_j\rangle, V_{Xj}) = p(V_{Xj}) = |\langle u_j|\psi\rangle|^2$ (Wigner, 1963), out of which only one *output* $V_{Xj'} \in \{V_{Xj}\}$ is somehow selected during an output-registration ("reduction"), with an output probability density $p(V_{Xj}) = |\langle u_j|\psi\rangle|^2$ that conserves the input distribution: the real physical succession commanded by the spacetime structure of the involved probability tree is violated, and the individual and statistical level of description are skipped, only the probabilistic level being expressed. On the other hand, this entails computability. But nothing hinders to supplement this formal scenario, by a representation (8), thereby restoring the physical succession as well as a full expression of all the involved descriptional levels.

The considerations of this section yield a new insight into the quantum mechanical transformation theory.

- *The relations (9)–(13) and (2′) permit us to form an intuitive notion concerning the physical characteristics of the processes that can lead to the quantum mechanical transformation theory; they yield a model for these.*

I have remarked before that the assumption of actualization of only *one* ontological property V_{Xj} of the studied entity, but one that produces *simultaneously* all the different considered qualifications ω^h_{jX} tied to M_X, leads to the same *formal* effect (5) as the more restrictive Kolmogorov assumption of ontological preexistence of such properties. Therefore, as long as we stay inside only *one* branch of the information tree $I\mathbf{T}(P_\psi)$ corresponding to the quantum mechanical probability tree $\mathbf{T}(P_\psi)$, Kolmogorov-type transformation relations *are* applicable inside quantum mechanics, though in this particular case also the outputs are factually created by the involved random phenomenon. In *this* rather superficial sense—and only in this—the Kolmogorov concept of probability can be regarded as simply a particular instance of a "more general" concept of probability involved in quantum mechanics and in the theory of information. But I mention and emphasize that it is reducing to try to express the relation between these two concepts of probability exclusively in terms of *formal* particularizations inside a more general *formal* representation: I have shown (Mugur-Schächter, 1993, pp. 94–95) that, in the order of increasingly complex conceptual elaborations, the quantum mechanical concept of probability is *prior* to Kolmogorov's concept, it is the *basic* concept of probability that emerges first and *out* of which the probabilities in Kolmogorov's sense are built by *meta*conceptualization, as a *meta*concept.

4.5. Concluding Remarks

I summarize the results. The representations (7), (8), and (9), (11), (2′) enrich both quantum mechanics and the information theory:

1. The two-level extension (7), (8) establishes a well-structured informational framework for the elaboration of the measurement theory of any attempt at a deterministic intrinsic interpretation of the quantum mechanical formalism.

2. The information theory is now endowed with the possibility of distinction between the microscopic and the macroscopic levels of description, and with explicit questions and formulations concerning an informational transformation theory.

3. The quantum mechanical transformation theory ceases to be mere posited blind algorithms, it acquires a model, and factual *significances* can be associated with it.

4. The quantum mechanical algorithms, in spite of their profound specificities, become in principle *comparable* with those of the informational representations.

5. OUTLOOK

In previous work (Mugur-Schächter, 1992*c*, 1993) I have shown that the quantum mechanical formalism has captured in it certain universal and very basic structures of conceptualization. Guided by the recognition of this fact, I have developed a "general syntax of *relativized* conceptualization" where the mists of false problems and paradoxes emanating from implicit false absolutes are cleaned away from the descriptions. The whole unending multiplicity of possible descriptional viewpoints is explicitly taken into account, and it generates a corresponding *coherent* and *hierarchical* unending multiplicity of relativized descriptions, each of which is crystal clear, and is connected to the others in a crystal clear way. Inside this general syntax of relativized conceptualization, I have identified the relativized form of the *most basic* sort of probabilistic conceptualization. And I have shown that *quantum probabilities are a particular instance of this form,* whereas Kolmogorov probabilities are the result of a *subsequent* probabilistic conceptualization, founded—implicitly—on the basic one.

In other works (Mugur-Schächter, 1980, 1992*c*; Mugur-Schächter and Hadjisavvas, 1982), I have *derived* the concept of informational entropy, whereas Shannon just *posited* it. The derivation involves a new functional, "the functional of opacity of a statistics with respect to the acting probability law," that defines mathematically the connection between a probability measure, its informational entropy, and all the statistics possible on the involved universe of elementary events. The opacity functional integrates the weak law of big numbers into a far more complex concept of converging evolution. And—*if it is relativized*—it endows the acting probability law and its informational entropy with a remakable *significance,* namely that of an "attractor" of all the various statistics that are possible on the involved universe of elementary events, toward a family of "relative metaforms" *encoded in the acting probability law and mathematically represented (in a global, mean sense) by the informational entropy of this law* (Mugur-Schächter, 1993). In this way, probabilities and information are deeply unified inside the general syntax of relativized conceptualization.

Consider now the results of this work.

From a mathematical point of view, the formalism of the quantum theory is far more powerful and precise than that of the information theory. Its computational capacities are outstanding. On the other hand, the theory of information is intuitive, and it has a quite remarkable *generality*: it applies to every conceivable change, regarded as a process by which an initial input signal (perturbation, process, etc.) produces an output sign (effect of any nature), by some interaction regarded as an information channel. In particular, it is possible to represent, in informational language, networks of *closed chains* of information systems where the last outputs are injected into the initial input source, entailing "self-organization." Now, the analyses from the last section of this work suggest the possibility of a symbiosis. *The specific performances of the quantum mechanical bra-ket formalism might come out to be transferable to the informational approach.* The messages emitted by information sources, represented by mere strings of symbols instead of functions, might come out to *always* admit of a representation as "message-vectors" *forming a vector space*. Correlatively, *any* channel might come out to admit, like the quantum measurement channels, of a *nonprobabilistic* representation, by somehow specifying for it a family of "message-eigenvectors" ("eigenstrings" of symbols) which characterize the channel *independently* of any specificity of this or that input source, and which *stay invariant by passage through that channel.* Such a representation could then be split by convenient particularizations, so as to distinguish between "dead" channels and "living" channels (Varela, 1989). The definition of the output *and* (in contradistinction to quantum mechanics) also the input probabilities, would have to be achieved via (a) a well-formulated principle of superposition of the effects (messages) of *interacting* information sources, (b) a principle of spectral decomposability of any message-vector, on the basis of eigenmessages introduced by any channel, and (c) for any given class of input sources and channels of a definite *nature*, specifically convenient algorithms for calculating the output and input probability laws in such a way that these laws, when estimated for any given interaction source, shall be correctly related to the separate laws estimated for the sources that interact inside that interaction source. In short, the informational algorithms might come out to accept reformulation in terms of a very general sort of "informational bra-ket algebra," yielding back the quantum formalism as only a particular realization (in which all the message-vectors and message-eigenvectors are represented by *functions* determined with the help of linear differential operators and equations, and the input probability laws are skipped). In order to exclude descriptional knots and mists tied to nonreferred, absolute formulations, the whole approach would have to be attempted inside the general syntax of relativized conceptualization.

So, on a still far horizon, I perceive the first contours of a general and radically relativized mathematical representation of the emergence and transmission of "forms," of *patterns* of any kind: patterns of inorganic or of organic matter relating a parcel of physical reality to another one, or relating matter to mind or mind to matter, or mind to mind; or relating patterns of behavior to . . . , etc.

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